Cambridge Assessment International Education

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4836874740

CAMBRIDGE INTERNATIONAL MATHEMATICS **0607/61**

Paper 6 Investigation and Modelling (Extended) **May/June 2024**

1 hour 40 minutes

You must answer on the question paper.

No additional materials are needed.

INSTRUCTIONS

- Answer both part **A** (Questions 1 to 7) and part **B** (Questions 8 to 13).
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- You should use a graphic display calculator where appropriate.
- You may use tracing paper.
- You must show all necessary working clearly, including sketches, to gain full marks for correct methods.
- In this paper you will be awarded marks for providing full reasons, examples and steps in your working to communicate your mathematics clearly and precisely.

INFORMATION

- The total mark for this paper is 60.
- The number of marks for each question or part question is shown in brackets [ ].

This document has **16** pages. Any blank pages are indicated.

[Turn over

Answer **both** parts **A** and **B**.

A INVESTIGATION (QUESTIONS 1 to 7)

SUMS OF POWERS (30 marks)

You are advised to spend no more than 50 minutes on this part.

This investigation looks at connections between the sum of the positive integers, $1+2+3+\ldots$, the sum of their squares, the sum of their cubes and the sum of their 5th powers.

Example $1+2+3+4+5+6+7+8+9+10+11+12=78$

The numbers can also be added using this method.

Step 1	Write down the first half of the numbers in a row.	1 2 3 … 6
Step 2	Write down the second half of the numbers underneath the first half but in reverse order.	12 11 10 … 7
Step 3	Add each column of two numbers to make a third row.	13 13 13 … 13
Step 4	Find the total of the numbers in the third row by writing the calculation as a multiplication.	$6\times13=78$

1 Use the method to complete the sum of the first 60 positive integers.
That is $1+2+3+\cdots+60=1830$.

1 2 3 ⋯ .....

.... ⋯ .....

.... ⋯ .....

..... × ..... = 1830

[3]

2 Use the method to calculate the sum of the first 128 positive integers.
That is $1+2+3+\ldots+128$.

..... [3]

3 Complete the table.
Use **Question 1**, **Question 2** and any patterns you notice.

	Number of positive integers, starting at 1	Multiplication	Sum
	12	6×13	78
	26	13×27	351
Question 1	60		1830
Question 2	128		
	204		20 910

[1]

4 $1+2+3+\ldots+n$ has n positive integers and its sum is T.

Find a formula for T in terms of n.

..... [2]

5 **(a)** Complete the table.

n	Sum of first n positive integers		Sum of first n square numbers		$\frac{S}{T}$ written as a fraction with denominator 3
	Calculation	Sum (T)	Calculation	Sum (S)	
1	1	1	1^2	1	$\frac{3}{3}$
2	$1+2$	3	1^2+2^2	5	$\frac{5}{3}$
3	$1+2+3$	6	$1^2+2^2+3^2$	14	$\frac{}{3}$
4	$1+2+3+4$	10	$1^2+2^2+3^2+4^2$		
5	$1+2+3+4+5$	15	$1^2+2^2+3^2+4^2+5^2$	55	

[3]

(b) Find an expression for $\frac{S}{T}$ in terms of n.

..... [3]

(c) The sum of the first 60 positive integers is 1830.

Find the sum of the first 60 square numbers.

..... [2]

(d) Use **Question 4** and **Question 5(b)** to help you find a formula for S in terms of n.
Write your answer as a single fraction.

..... [1]

6 **(a)** Complete the table.

Sum of first n positive integers		Sum of first n cube numbers	
Calculation	Sum (T)	Calculation	Sum (C)
$1+2$	3	1^3+2^3	9
$1+2+3$	6	$1^3+2^3+3^3$	36
$1+2+3+4$	10	$1^3+2^3+3^3+4^3$	100
$1+2+3+4+5$	15	$1^3+2^3+3^3+4^3+5^3$	225
$1+2+3+4+5+6$		$1^3+2^3+3^3+4^3+5^3+6^3$	

[1]

(b) Write a formula for C in terms of T.

..... [1]

(c) $1+2+3+\ldots+60=1830$

Calculate $1^3+2^3+3^3+\ldots+60^3$.

..... [2]

7 **(a)** Complete the table.
Use **Question 6(a)** to help you.

n	Sum of first n cube numbers		Sum of first n 5th powers		$\frac{F}{C}$ as a fraction with denominator 3
	Calculation	Sum (C)	Calculation	Sum (F)	
1	1^3	1	1^5	1	$1 = \frac{3}{3}$
2	$1^3 + 2^3$	9	$1^5 + 2^5$	33	$\frac{33}{9} = \frac{11}{3}$
3	$1^3 + 2^3 + 3^3$	36	$1^5 + 2^5 + 3^5$	276	$\frac{276}{36} = \frac{23}{3}$
4	$1^3 + 2^3 + 3^3 + 4^3$	100	$1^5 + 2^5 + 3^5 + 4^5$	1300	
5	$1^3 + 2^3 + 3^3 + 4^3 + 5^3$	225	$1^5 + 2^5 + 3^5 + 4^5 + 5^5$	4425	$\frac{4425}{225} = \frac{59}{3}$
6	$1^3 + 2^3 + 3^3 + 4^3 + 5^3 + 6^3$		$1^5 + 2^5 + 3^5 + 4^5 + 5^5 + 6^5$		

[2]

(b) The fraction $\frac{F}{C}$ is written with a denominator of 3.

Find an expression for the numerator in terms of n.

..... [4]

(c) $1+2+3+\ldots+60 = 1830$

Calculate $1^5+2^5+3^5+\ldots+60^5$.

Write down all the numbers on your calculator display.

..... [2]

The modelling starts on the next page.

B MODELLING (QUESTIONS 8 to 13)

INCOME INEQUALITY (30 marks)

You are advised to spend no more than 50 minutes on this part.

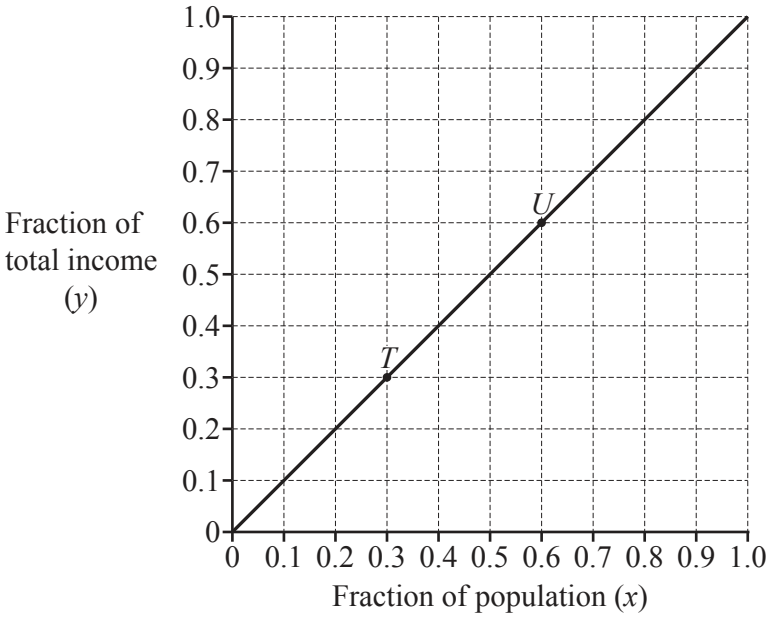
This task looks at a model to measure the spread of income within the population of a country.

In this task, x is the decimal fraction of the population of a country
and y is the decimal fraction of the total income for the country.

Examples
When $x = 1$ this is the total population of a country.
When $x = 0.5$ this is half of the population of a country.

When $y = 1$ this is the total income for the country.
When $y = 0.5$ this is half of the total income for the country.

The graph shows how the total income of a country is shared among the population.



The point T shows that 0.3 of the population of a country earn 0.3 of the total income.
The point U shows that 0.6 of the population of a country earn 0.6 of the total income.

8 **(a)** Mark a point on the line and label it Z.
Complete this statement for your point.

The point Z shows that ..... of the population of a country earn ..... of the total income.

[1]

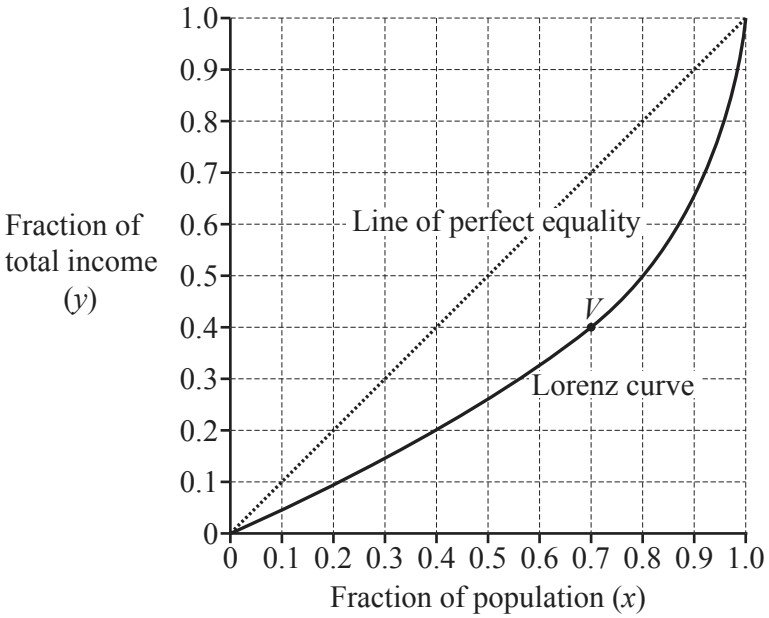
(b) For this graph there is perfect equality because the income is shared equally among the population of a country.
The graph shows the *line of perfect equality*.

Write down the equation of the line.

..... [1]

9 In reality, the total income of a country is not shared equally, so there is income inequality.
In 1905 the American economist Max Lorenz invented the *Lorenz curve* to show income inequality.
A Lorenz curve is always on or below the line of perfect equality.

The graph shows the Lorenz curve for one country.



The point *V* on this Lorenz curve shows that the poorest 0.7 of the population only earn 0.4 of the total income.

(a) Use another point on the Lorenz curve to make a similar statement.

.....

..... [1]

(b) From the statement for point *V* we can also say that the richest 0.3 of the population earn 0.6 of the total income.

(i) Write calculations to show why the values in this statement are correct.

.....

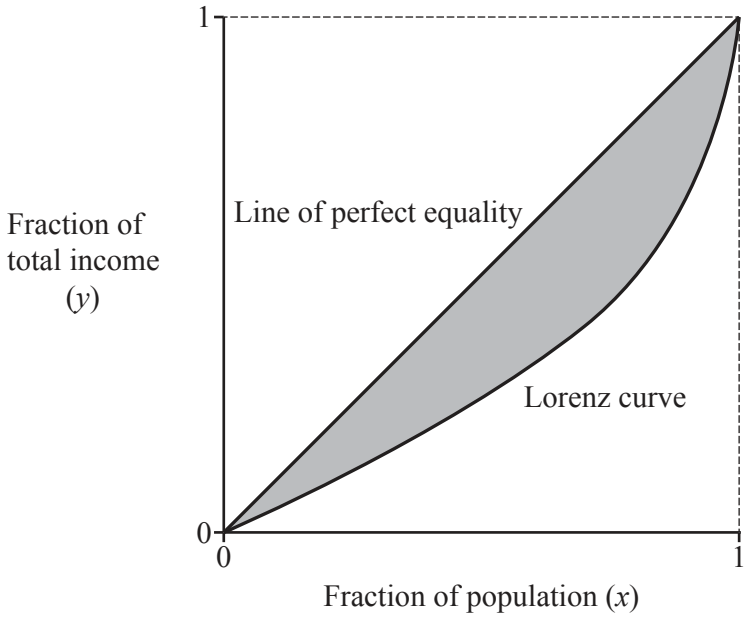
..... [1]

(ii) Write a similar statement for the point that you chose in **part (a)**.

.....

..... [1]

10 In 1912 the Italian statistician Corrado Gini invented the *Gini coefficient*.
The Gini coefficient measures how much income inequality there is in a country.



The Gini coefficient is **two times** the shaded area between the line of perfect equality and the Lorenz curve.
This means that the greater the shaded area, the greater the income inequality.

(a) When there is perfect equality of income write down the Gini coefficient.

..... [1]

(b) When the income inequality is a maximum, the shaded area will be as large as possible.

Find the Gini coefficient when there is maximum inequality of income.

..... [2]

11 Mei uses these steps to model the Gini coefficient for the curve in **Question 9**.

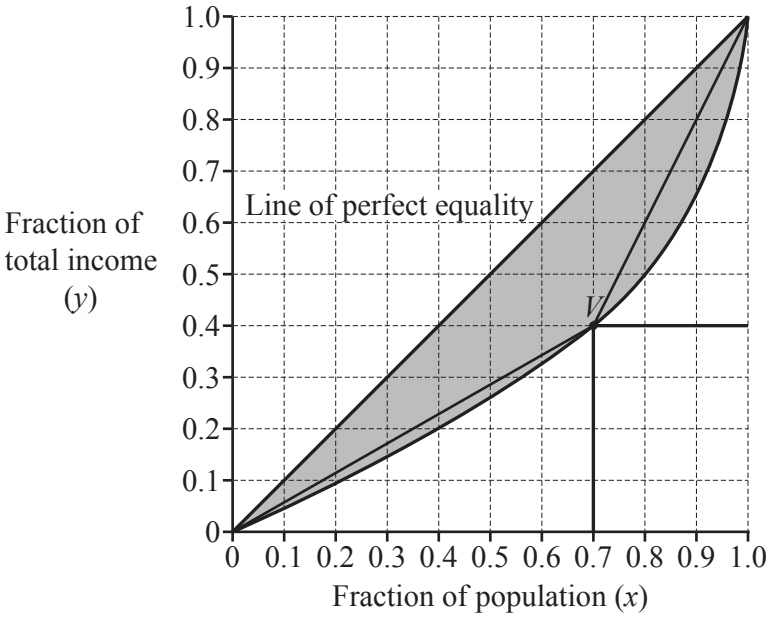
Step 1: Plot point V from **Question 9** on the Lorenz curve.

Step 2: Approximate the area below the curve with a rectangle and two triangles.

Step 3: Calculate the total area of the rectangle and the two triangles. This is T.

Step 4: Calculate the shaded area by subtracting T from the area of the large triangle with vertices (0, 0), (1, 1) and (1, 0).

Step 5: Multiply the result by 2.

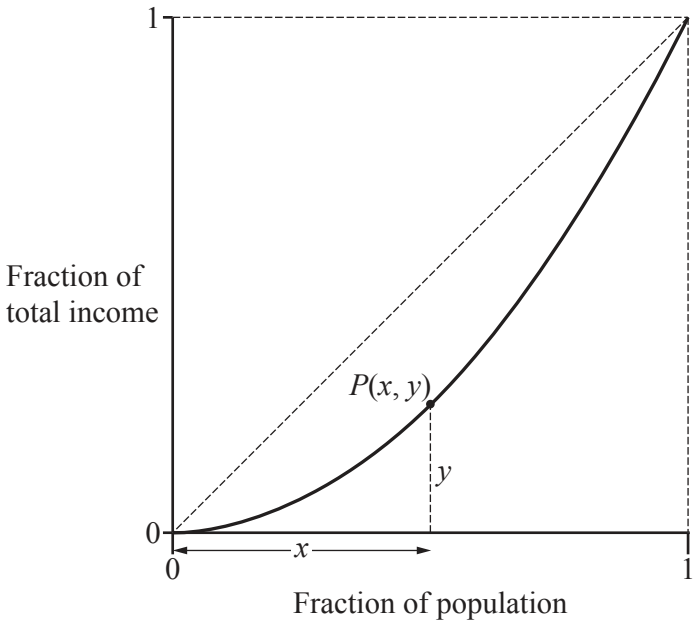


Steps 1 and 2 in Mei's model have been done for you on the graph.

Do Steps 3, 4 and 5 to calculate her approximation of the Gini coefficient.

..... [6]

12 Mei decides to use a point $P(x, y)$ in Step 1 of her model.



(a) Find the area in Step 3 of her model as an expression without brackets in terms of x and y.

..... [5]

(b) Do steps 4 and 5 in her model to show that her approximation, G, for the Gini coefficient is $x - y$.

[2]

(c) Give a reason why G will be smaller than the actual Gini coefficient.

..... [1]

13 Mei knows that her approximation, $G = x - y$, is always smaller than the actual Gini coefficient. So, to make her model as accurate as possible, she takes the maximum value of $G = x - y$.

(a) For Country A, the equation of the Lorenz curve is $y = x^2$.

Find Mei's most accurate estimate for the Gini coefficient by finding the maximum of $G = x - x^2$.

..... [2]

(b) (i) Write down the coordinates of the two points that must be on every Lorenz curve.

..... [1]

(ii) For Country B, the equation of the Lorenz curve is $y = a - \sqrt{1-x}$ where a is a constant.

Use **part (i)** to find the value of a.

..... [1]

(iii) Sketch the graph of $G = x - y$ for Country B.

[2]

(c) Use the most accurate estimates in Mei's model of the Gini coefficient to compare income inequality in Country A and Country B.

..... [2]

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